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Theory of Clarifier Operation. II. Hindered Settling of Flocculating Systems in Rectangular Clarifiers

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Abstract

The operation of rectangular clarifiers is simulated by means of the continuity equations. The model permits the examination of hindered settling of solids which are undergoing flocculation and floc disruption. The steady-state solution requires very modest amounts of computer time and also lends itself to the analysis of quiescent settling in jar tests.

INTRODUCTION

We recently examined the modeling of clarifier operation by use of the continuity equations for hindered settling of flocculating suspensions; the case of quiescent settling such as is used in jar tests was analyzed (1). The relevant literature was reviewed in that paper, referred to henceforth as I. The use of the continuity equations, into which one inserts terms corresponding to the formation of composite particles by flocculation and terms corresponding to the disruption of composite particles by viscous drag forces, permits one to construct mathematical models of the various types of clarifiers which are quite realistic and which are easily within the capabilities of even rather modest electronic computers.

One of the most commonly used types of clarifiers is the rectangular clarifier in which liquid is admitted at one end of a rectangular tank, slowly moves across the tank (during which process the solids settle out), and is discharged at the other end of the tank. Inlet structures may be

provided to reduce turbulence in the tank by breaking the flow of the entering liquid. Facilities for removal of the settled sludge are necessary, and discharge of clarified effluent may be via a weir of some sort.

We present here a mathematical model for the description of the steady-state operation of rectangular clarifiers in the hindered settling regime with flocculating particles.

ANALYSIS

We assume for our model a collection of particles formed by flocculation from unit elementary particles, disrupted by viscous drag forces, moved in the horizontal (x) direction by the flow of liquid through the clarifier from left to right, and moved downward ($-y$) by the interplay between gravity and viscous drag. We include terms representing diffusive mixing in the x and y directions. The continuity equation for n -particles (consisting of n elementary particles) is

$$\frac{\partial c_n}{\partial t}(x, y, t) = -\nabla \cdot (v'_n c_n) + \frac{\partial}{\partial x} \left(D_x \frac{\partial c_n}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_y \frac{\partial c_n}{\partial y} \right) + F_n[\mathbf{c}(x, y, t)],$$

$$n = 1, 2, \dots, N \quad (1)$$

where t = time

c_n = number density of n -particles at (x, y, t)

v'_n = velocity in the laboratory frame of reference of an n -particle at (x, y, t)

D_x, D_y = effective diffusion constants

$\mathbf{c}(x, y, t) = [c_1, c_2, \dots, c_N(x, y, t)]$

F_n = flocculation and disruption terms

N = largest composite particle permitted

As before (1), we take the flocculation and floc disruption terms to be given by

$$F_n = \sum_{j=1}^{\lfloor n/2 \rfloor} c_j c_{n-j} |v_j - v_{n-j}| \pi (r_j + r_{n-j})^2 - \sum_{j=1}^{N-n} c_j c_n |v_j - v_n| \pi (r_j + r_n)^2$$

$$+ \sum_{j=n+1}^N k_{n,j-n}^j c_j (1 + \delta_{n,j-n}) - \sum_{j=1}^{\lfloor n/2 \rfloor} k_{j,n-j}^n c_n \quad (2)$$

where $\delta_{ij} = 0$ if $i \neq j$; $= 1$ if $i = j$

$\lfloor n/2 \rfloor$ = largest integer $\leq n/2$

v_k = velocity of a k -particle relative to the surrounding liquid

r_k = radius of a k -particle, assumed spherical, = $(3kV_1/4\pi)^{1/3}$

V_1 = volume of an elementary particle

V_k = volume of k -particle, = kV_1

We calculate the v_k as follows. We take Vand's formula for the viscosity of a slurry (2)

$$\eta = \eta_0 \exp \left[\frac{2.5C + 2.7C^2}{1 - 0.609C} \right] \quad (3)$$

where C is the volume fraction of solids in the slurry at the point of interest, given by

$$C = \sum_{n=1}^N c_n(x, y, t) V_n \quad (4)$$

and η_0 is the viscosity of the pure liquid. The density of the slurry is given by

$$\rho_{sl} = \rho_s C + \rho_l(1 - C) \quad (5)$$

where ρ_s = density of solid

ρ_l = liquid density

We define $\Delta\rho$ to be $\rho_s - \rho_{sl}$, which yields

$$\Delta\rho = (\rho_s - \rho_l)(1 - C) \quad (6)$$

The y -velocity of a k -particle relative to the surrounding liquid is then given by

$$-v_k = u_k = \frac{2g(\Delta\rho)rk^2}{9\eta} \left[1 + \frac{1}{4} (\rho_{sl}r_k u_k / 2\eta)^{1/2} + 0.34\rho_{sl}r_k u_k \right]^{-1} \quad (7)$$

obtained from Fair, Geyer, and Okun (3). Here g is the gravitational constant. The y -velocity of a k -particle relative to the laboratory is given by (1)

$$v'_k = v_k - \sum_{n=1}^N v_n c_n V_n \quad (8)$$

We take the following expression for the rate constant for floc disruption:

$$k_{n,j-n}^j = \kappa \frac{jj!}{n!(j-n)!} \frac{[N/2]!(N - [N/2])!}{NN!} \quad (9)$$

where κ is just a proportionality constant (1), and $[N/2]$ is the greatest integer less than or equal to $N/2$.

The strongly nonlinear character of Eq. (1) precludes its solution by analytical means; we therefore represent the spatial dependence of the $c_n(x, y, t)$ by means of a discrete mesh of points. We let

$$c(n, p, q, t) = c_n[(p - \frac{1}{2})\Delta x, (q - \frac{1}{2})\Delta y, t] \quad (10)$$

and approximate Eq. (1) as follows.

$$\begin{aligned} \frac{\partial c}{\partial t}(n, p, q, t) &= \frac{1}{\Delta x}[-v_x(n, p, q, t)c(n, p, q, t) + v_x(n, p - 1, q, t)c(n, p - 1, q, t)] \\ &\quad + \frac{1}{\Delta y}[-v'_y(n, p, q + 1, t)c(n, p, q + 1, t) + v'_y(n, p, q, t)c(n, p, q, t)] \\ &\quad + \frac{D_x}{\Delta x^2}[c(n, p + 1, q, t) - 2c(n, p, q, t) + c(n, p - 1, q, t)] \\ &\quad + \frac{D_y}{\Delta y^2}[c(n, p, q + 1, t) - 2c(n, p, q, t) + c(n, p, q - 1, t)] \\ &\quad + F_n[c(p, q, t)] \end{aligned} \quad (11)$$

Solution of Eq. (11) with appropriate initial and boundary conditions and a substantial number of particle sizes represents a possible but rather formidable problem in terms of computer time. We therefore address ourselves to two simpler but still quite realistic variants: (a) the time-dependent problem for a nonflocculating monodisperse system, and (b) the steady-state problem for the polydisperse flocculating system. We also assume that v_x is a constant—that we have plug flow across the clarifier, and we drop the diffusive mixing terms.

The equations for the time-dependent monodisperse system with the above constraints become

$$\begin{aligned} \frac{dc}{dt}(p, q, t) &= \frac{v_x}{\Delta x}[c(p - 1, q, t) - c(p, q, t)] \\ &\quad + \frac{1}{\Delta y}[-v'(p, q + 1, t)c(p, q + 1, t) + v'(p, q, t)c(p, q, t)] \end{aligned} \quad (12)$$

with some obvious simplifications in notation. Along the top of the

clarifier $q = Q$, and we have

$$\frac{dc}{dt}(p, Q, t) = \frac{v_x}{\Delta x} [c(p - 1, Q, t) - c(p, Q, t)] + \frac{1}{\Delta y} [v'(p, Q, t)c(p, Q, t)] \quad (13)$$

on noting that there is no flux into these cells from above. Along the bottom $q = 1$, and

$$\frac{dc}{dt}(p, 1, t) = \frac{v_x}{\Delta x} [c(p - 1, 1, t) - c(p, 1, t)] + \frac{1}{\Delta y} [-v'(p, 2, t)c(p, 2, t)] \quad (14)$$

Along the left side of the clarifier $p = 1$, and

$$\begin{aligned} \frac{dc}{dt}(1, q, t) = & \frac{v_x}{\Delta x} [c^\circ(q, t) - c(1, q, t)] \\ & + \frac{1}{\Delta y} [-v'(1, q + 1, t)c(1, q + 1, t) + v'(1, q, t)c(1, q, t)] \end{aligned} \quad (15)$$

where $c^\circ(q, t)$ gives the influent concentration. At the top left corner, $p = 1, q = Q$, and

$$\frac{dc}{dt}(1, Q, t) = \frac{v_x}{\Delta x} [c^\circ(Q, t) - c(1, Q, t)] + \frac{1}{\Delta y} [v'(1, Q, t)c(1, Q, t)] \quad (16)$$

This set of equations, Eqs. (12)–(16), we write in abbreviated notation as

$$\frac{\partial c}{\partial t}(p, q, t) = f_{pq}[\hat{c}(t)], \quad \begin{matrix} p = 1, P \\ q = 1, Q \end{matrix} \quad (17)$$

where $\hat{c}(t)$ represents the set $c(i, j, t)$, for all i, j . We integrate Eq. (17) by means of a predictor-corrector method, using as our starting formulas:

predictor

$$c^*(p, q, \Delta t) = c(p, q, 0) + \Delta t f_{pq}[\hat{c}(0)] \quad (18)$$

corrector

$$c(p, q, \Delta t) = c(p, q, 0) + \frac{\Delta t}{2} \{f_{pq}[\hat{c}(0)] + f_{pq}[c^*(\Delta t)]\} \quad (19)$$

Generally we use the algorithm

$$c^*(p, q, t + \Delta t) = c(p, q, t - \Delta t) + 2\Delta t f_{pq}[\hat{c}(t)] \quad (20)$$

$$c(p, q, t + \Delta t) = c(p, q, t) + \frac{\Delta t}{2} \{f_{pq}[\hat{c}(t)] + f_{pq}[\hat{c}(t + \Delta t)]\} \quad (21)$$

The principal purpose of the computer program written to solve this system was to examine the approach of the clarifier to steady-state conditions after start-up. Several runs were made in which the concentrations of solids at 400 points in the clarifier were printed at 10-sec intervals (clarifier time) so that the time-dependence of the concentration profiles could be examined. One observes the solids front moving across the clarifier and falling from left to right as the particles settle; the time constant for the approach to steady-state conditions was about x_l/v_x , the time required for the water to transit the clarifier, as one would expect. The enormous quantity of data generated did not lend itself well to summary presentation, so we calculated the ratio of effluent solids flux to influent solids flux,

$$E(t) = \sum_{q=2}^Q c(P, q, t) / \sum_{q=1}^Q c(1, q, t) \quad (22)$$

in which we assume that the effluent from the lowest compartment on the right side of the clarifier is discharged as sludge.

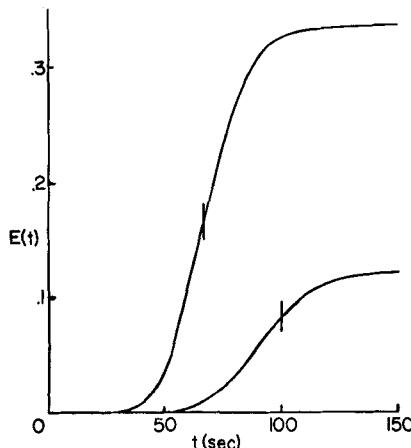


FIG. 1. Clarifier efficiency factor $E(t)$ as a function of time after start-up. The system is monodisperse; $\eta_0 = 0.01$ P, $\rho_s = 1.25$ g/cm³, $\rho_l = 1.00$ g/cm³, $r_1 = 0.01$, $x_l = 100$ cm, $y_i = 50$ cm, $v_x = 1.50$ (upper curve) or 1.00 (lower curve) cm/sec, $c_0 = 0.01$ cm⁻³, $\Delta t = 1$ sec, $n_x = n_y = 20$.

The dependence of $E(t)$ on v_x , the linear velocity of water through the clarifier, is shown in Fig. 1. The vertical bars show the time at which solids would reach the effluent if pure plug flow were operative. The duration of the time interval during which $E(t)$ increases from essentially zero to essentially its asymptotic value decreases as one increases the values of P and Q (decreasing the size of the compartments into which the clarifier is partitioned). We see the expected increase in clarifier efficiency as v_x is decreased. Figure 2 exhibits the effect of particle density on clarifier efficiency, and shows very marked improvement in $E(t)$ as particle density increases, as one would expect. Each of these curves required a little less than 5 min of XDS Sigma 7 computer time.

We next address ourselves to the time-independent, steady-state case. We take Eq. (11) as our starting point, drop the diffusive terms again, and assume that v_x is constant. Solving for $c(n, p, q)$ then yields

$$c(n, p, q) = \left\{ \frac{v_x}{\Delta x} c(n, p - 1, q) - \frac{v'(n, p, q + 1)c(n, p, q + 1)}{\Delta y} \right. \\ \left. + F_n[c(p, q)] \right\} \left/ \left[\frac{v_x}{\Delta x} - \frac{v'(n, p, q)}{\Delta y} \right] \right., \\ p = 2, 3, \dots, P \\ q = Q - 1, Q - 2, \dots, 3, 2 \quad (23)$$

When $q = Q$ (along the upper boundary),

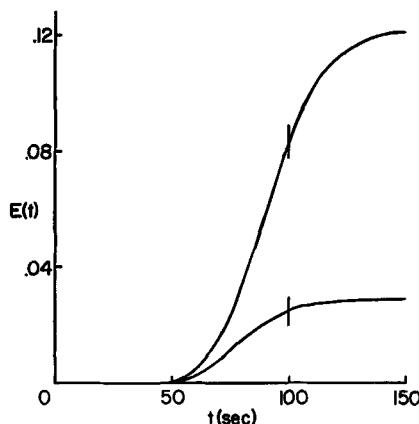


FIG. 2. Clarifier efficiency factor as a function of time after start-up. Parameters are as in Fig. 1 except that $v_x = 100$ cm/sec for both curves, and $\rho_s = 1.25$ (upper curve) and 1.35 (lower curve) g/cm³.

$$c(n, p, Q) = \left\{ \frac{v_x}{\Delta x} c(n, p - 1, Q) + F_n[c(p, Q)] \right\} \left/ \left[\frac{v_x}{\Delta x} - \frac{v'(n, p, Q)}{\Delta y} \right] \right. \quad (24)$$

When $q = 1$ (along the lower boundary),

$$c(n, p, 1) = c(n, p - 1, 1) + \left\{ \frac{-v'(n, p, 2)}{\Delta y} c(n, p, 2) + F_n[c(p, 1)] \right\} \frac{\Delta x}{v_x} \quad (25)$$

It is necessary to iterate Eqs. (23)–(25) a couple of times, since $v'(n, p, q)$ and F_n , which appear on the right side of the equations, both depend on $c(n, p, q)$. Three iterations were sufficient in all of the cases we tested. Our boundary conditions are that no material enters through the top of the clarifier or leaves through the bottom, and that constant solids feed concentration (n particles per cm^3) are maintained on the left side of the clarifier,

$$c(n, 1, q) = c_0 n^{-k} \quad (26)$$

where k was taken equal to 2.

Our computer program for the steady-state case prints out the volume fraction of solids in each compartment into which the clarifier is partitioned, again an embarrassingly large quantity of data even when the clarifier is partitioned into only 400 compartments. Again we calculate a clarifier efficiency factor,

$$E = \sum_{n=1}^N \sum_{q=2}^Q c(n, P, q) V(n) / \sum_{n=1}^N \sum_{q=1}^Q c(n, 1, q) V(n) \quad (27)$$

the ratio of the effluent solids volume fraction to the influent solids volume fraction. We again assume that the effluent from the bottom compartment on the right-hand end of the clarifier is discharged as sludge. A single run took approximately 10 sec of XDS Sigma 7 computer time when particles of five different sizes were allowed; with nine different sizes of particles, about 25 sec were required.

RESULTS

The dependence of clarifier efficiency on r_1 , the radius of the particles of minimum size, is given in Table 1. The results are certainly as one would expect— E decreases drastically with increasing r_1 . They also demonstrate that, even though flocculation is permitted to occur in the system, the size of the “fines”—the smallest particles present—is very important in

determining clarifier performance; the smaller the fines, the poorer the quality of the effluent. Qualitatively, this result is well-known to anyone familiar with the operation of clarifiers following biological waste treatment; the model permits us to make a more quantitative assessment of the effect in flocculating systems. In Table 1 and subsequently, TISC stands for total influent solids concentration, the volume fraction of total suspended solids in the influent.

Table 2 shows the dependence of clarifier efficiency on the velocity of throughput, and clearly exhibits the very sharp increase in effluent solids which one expects when the flow rate through the clarifier is increased to too large a value. With solids having the characteristics simulated here, our simulated clarifier evidently operates at 95% efficiency at a linear flow rate of about 0.85 cm/sec.

Solids density, ρ_s , displays the anticipated strong influence on clarifier efficiency, as seen in Table 3. Changing the solids density from 1.050 to

TABLE 1
Dependence of Clarifier Efficiency Function E on Elementary Particle Radius
 r_1^a

$r_1 \times 10^2$ (cm)	TISC	E
1	1.185×10^{-4}	0.5963
2	9.48×10^{-4}	0.0847
3	3.20×10^{-3}	7.67×10^{-3}
4	7.58×10^{-3}	1.987×10^{-4}

^a Other system parameters are: $\eta_0 = 0.01$ P, $\rho_s = 1.05$ g/cm³, $\rho_l = 1.00$ g/cm³, $x_l = 100$ cm, $y_l = 50$ cm, $v_x = 1.00$ cm/sec, $\kappa = 0.10$ sec⁻¹, $c_0 = 10$ cm⁻³, $n_x = n_y = 20$, $k = 2$, $N = 9$.

TABLE 2
Dependence of Clarifier Efficiency Function on Horizontal Linear Flow Velocity v_x^a

v_x (cm/sec)	E
0.25	2.136×10^{-5}
0.50	8.65×10^{-3}
0.75	3.776×10^{-2}
1.25	0.1351
1.50	0.3069

^a Other system parameters are: $\eta_0 = 0.01$ P, $\rho_s = 1.05$ g/cm³, $\rho_l = 1.00$ g/cm³, $r_1 = 0.02$, $x_l = 100$ cm, $y_l = 50$ cm, $\kappa = 0.10$ sec⁻¹, $c_0 = 10$ cm⁻³, $n_x = n_y = 20$, $k = 2$, $N = 9$, TISC = 0.948×10^{-3} .

1.025 g/cm³ results in a decrease in our clarifier's efficiency from about 92 to 73%. Biological flocs are only very slightly more dense than water, hence settle extremely slowly. If one could find conditions under which these flocs would attach to fine sand particles, one might be able to increase their effective density with quite beneficial results.

The influent floc particle concentrations used in these simulations were calculated from Eq. (26), where k was set equal to 2 throughout. In Table 4 we examine the dependence of clarifier efficiency on solids loading as controlled by the variation of c_0 in Eq. (26). Increasing the solids concentration favors efficient operation by speeding the formation of large, rapidly settling particles by flocculation, a second-order process. On the other hand, it hinders efficient operation by increasing the effective viscosity of the slurry (see Eq. 3) and by decreasing the density difference between the settling particles and the slurry (Eq. 6). For the conditions we have simulated here, it is evident that increased solids concentrations do enhance the rate of settling.

TABLE 3
Dependence of Clarifier Efficiency Function on Solids Density ρ_s^a

ρ_s (g/cm ³)	E
1.025	0.2703
1.05	0.0847
1.10	1.168×10^{-2}
1.25	1.178×10^{-5}

^a Other system parameters are: $\eta_0 = 0.01$ P, $\rho_t = 1.00$ g/cm³, $r_1 = 0.02$, $x_1 = 100$ cm, $y_1 = 50$ cm, $v_x = 1.00$ cm/sec, $\kappa = 0.10$ sec⁻¹, $c_0 = 10$ cm⁻³, $n_x = n_y = 20$, $k = 2$, $N = 9$, TISC = 0.948×10^{-3} .

TABLE 4
Dependence of Clarifier Efficiency Function on Solids Concentration^a

c_0 (cm ⁻³)	TISC	E
1.0	9.48×10^{-5}	0.1402
5.0	4.74×10^{-4}	0.1284
10	9.48×10^{-4}	0.1164
20	1.90×10^{-3}	0.0653

^a Other system parameters are: $\eta_0 = 0.01$ P, $\rho_s = 1.05$ g/cm³, $\rho_t = 1.00$ g/cm³, $r_1 = 0.02$, $x_1 = 100$ cm, $y_1 = 50$ cm, $v_x = 1.00$ cm/sec, $\kappa = 10^{-3}$ sec⁻¹, $n_x = n_y = 20$, $k = 2$, $N = 5$.

Table 5 shows the dependence of clarifier efficiency on the maximum number of elementary particles which are permitted to aggregate together to form a composite floc particle. The (hardly surprising) result is that the weighting of the size distribution toward the larger particles results in a substantial increase in clarifier efficiency. We note that with this model it becomes a very simple (and cheap) matter to examine in detail the effects of particle size distribution on the performance of a clarifier. The initial size distribution function we have chosen, $c_0 n^{-2}$, can easily have the exponent of n varied as desired, or, by changing three cards in the program, another function could easily be substituted for the one which we have chosen more or less arbitrarily. One presumes that at higher values of the TISC the initial particle size distribution would become less influential, since the rate of coagulation of the particles, a second-order process, would be increased and the bulk of the settling would occur from a near-equilibrium distribution of particle sizes.

Table 6 shows the effect of varying the floc disruption constants; κ is the scale factor for these, indicated in Eq. (9). We see the expected deterioration in clarifier performance as the floc disruption rates are increased, but the effect is rather small. Increasing κ to about 4 sec^{-1} results in computational disaster, as the rates of break-up of the large particles become so large that we must choose values of Δx and Δy substantially smaller than those used here (5 and 2.5 cm, respectively, in a clarifier 100 cm long by 50 cm deep) in order to avoid negative or wildly oscillating concentrations. Evidently clarifier efficiency is determined primarily by the concentrations of the smaller particles (1-, 2-, and perhaps 3-particles), and the concentrations of these are relatively little affected even when the rate of break-up of 9-particles to form 4- and 5-

TABLE 5
Dependence of Clarifier Efficiency Function on Maximum Aggregate Size, N^a

N	TISC $\times 10^5$	E
1	3.35	0.2760
3	6.14	0.1750
5	7.65	0.1402
7	8.69	0.1228
9	9.48	0.1121

^a Other system parameters are: $\eta_0 = 0.01 \text{ P}$, $\rho_s = 1.05 \text{ g/cm}^3$, $\rho_l = 1.00 \text{ g/cm}^3$, $r_1 = 0.02$, $x_l = 100 \text{ cm}$, $y_l = 50 \text{ cm}$, $v_x = 1.00 \text{ cm/sec}$, $\kappa = 10^{-3} \text{ sec}^{-1}$, $c_0 = 1.0 \text{ cm}^{-3}$, $n_x = n_y = 20$, $k = 2$.

TABLE 6

Dependence of Clarifier Efficiency Function on the Floc Disruption Scale Factor κ^a

κ (sec $^{-1}$)	$E \times 10^2$
10^{-3}	8.47
10^{-2}	8.47
0.10	8.47
0.30	8.47
0.60	8.47
2.0	8.61
4.0	8.85

^a Other system parameters are: $\eta_0 = 0.01$ P, $\rho_s = 1.05$ g/cm 3 , $\rho_l = 1.00$ g/cm 3 , $r_1 = 0.02$, $x_l = 100$ cm, $y_l = 50$ cm, $v_x = 1.00$ cm/sec, $c_0 = 10$ cm $^{-3}$, $n_x = n_y = 20$, $k = 2$, $N = 9$, TISC = 9.48×10^{-4} .

particles is large enough to cause computational problems. This may be an artifact resulting from the functional form of our floc disruption constants. These are such that a large n -particle is much more likely to be broken up into two roughly equally sized pieces than into an $(n - 1)$ -particle and a 1-particle, and we do not know if this is in fact realistic.

There are a couple of other criticisms which can be made about the model. First, as the particles fall they are dissipating power in the suspension, which must result in small-scale turbulences. These turbulences should result in an increased frequency of particle-particle collisions above the estimate given by the first two sums in Eq. (2), which assume that the only mechanism leading to collisions is the differential rate of fall of the particles in a completely quiescent solution. This effect should increase with increasing solids concentrations as long as the viscosity and slurry density are not strongly affected.

A second criticism is that we have failed to include the diffusive terms present in Eqs. (1) and (11). Inclusion of these explicitly makes the computations much more time-consuming, necessitating the use of several iterations over the entire mesh of points. Also, the diffusive terms can be taken into account in a more or less empirical way by adjusting the spacing of the points in the mesh covering the clarifier—the values of Δx and Δy . Since we have no good way of estimating the diffusion constants D_x and D_y , we did not feel that additional expense was warranted.

We note that this program may be used to examine quiescent settling such as occurs in jar tests simply by making the correspondence $x/v_x = t$. In this way one can use the results of jar tests to obtain a set of parameters

which adequately characterize the influent to be clarified, and then use these parameters and information on maximum flow rates and desired effluent quality to design a clarifier meeting the desired specifications by means of the computer simulator.

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